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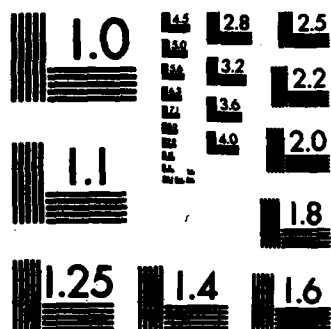
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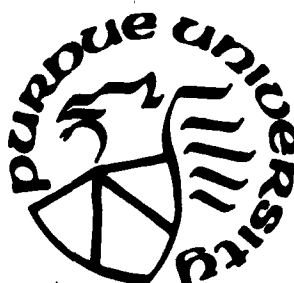
ISOTONIC PROCEDURES FOR SELECTING POPULATIONS  
BETTER THAN A STANDARD:  
TWO-PARAMETER EXPONENTIAL DISTRIBUTIONS\*

by

Shanti S. Gupta and Lii-Yuh Leu  
Purdue University

Technical Report #83-46

**PURDUE UNIVERSITY**



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Abstract

The problem of selecting populations, from two-parameter exponential populations, which are better than a standard under an ordering prior is investigated. If the negative exponential distribution is the model for lifetime, then the problem is to select all those populations for which the guarantee lifetimes are larger than that of a standard. Comparisons of these procedures based on the expected number of bad populations in the selected subset is investigated. Tables of associated constants for the proposed procedures are given in Table I through Table IV.

**Key words:** Isotonic procedures, selection procedures, standard, negative exponential, guarantee time, subset selection, simple ordering prior.



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1. Introduction

The problem of selecting populations better than a standard under an ordering prior has been considered by Gupta and Yang (1981) for the normal means problem and by Gupta and Huang (1982) for the binomial parameters problem. In this paper we consider the case of two-parameter exponential populations for which an interest lies in comparing location parameters (guarantee times).

In Section 2, notations and definitions used in this paper are introduced. Isotonic selection procedures are considered in Section 3, according to the control parameter and the common scale parameter which may be known or unknown. In Section 4, some other procedures for this problem are also considered. Comparisons of these procedures based on the expected number of bad populations in the selected subset is investigated. Tables of associated constants for the proposed procedures are given in Table I through Table IV.

2. Notations and Definitions

Let  $E(\mu, \theta)$  denote the two-parameter exponential distribution with probability density function

$$(2.1) \quad f(x; \mu, \theta) = \begin{cases} \theta^{-1} \exp\{-\theta^{-1}(x-\mu)\}, & \text{if } x > \mu \\ 0 & \text{if } x \leq \mu \end{cases}$$

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where  $-\infty < \mu < \infty$  and  $\theta > 0$ . The parameter  $\mu$  is called the guarantee time and  $\theta$  is the scale parameter which in this case is the standard deviation.

Suppose that  $\pi_0, \pi_1, \dots, \pi_k$  are  $(k+1)$  independent populations. It is assumed that the observations from  $\pi_i$  follow a  $E(\mu_i, \theta)$  distribution,  $i = 0, 1, \dots, k$ . The guarantee time is the parameter of interest. It is assumed that  $\mu_1 \leq \mu_2 \leq \dots \leq \mu_k$ ; however, the true values of these  $\mu_i$ 's are not known. We consider  $\pi_0$  as a control (or standard). We say that population  $\pi_i$  is "good" if  $\mu_i \geq \mu_0$ . Our goal is to select a subset of these  $k$  populations so that all "good" populations are included in the selected subset.

Let  $\Omega = \{\mu = (\mu_0, \mu_1, \dots, \mu_k) | -\infty < \mu_1 \leq \mu_2 \leq \dots \leq \mu_k < \infty, -\infty < \mu_0 < \infty\}$  be the parameter space. Let us denote the sets  $a_i = \{i, i+1, \dots, k\}$ ,  $1 \leq i \leq k$  and  $a_0 = \phi$  (the empty set). If action  $a_i$  is taken, it means the subset  $\{\pi_i, \pi_{i+1}, \dots, \pi_k\}$  of the  $k$  populations is selected. Since by our assumption  $\mu_i$ 's are ordered according to an ascending ordering prior, it is, therefore, appropriate to restrict our attention to the action space  $G = \{a_0, a_1, \dots, a_k\}$ . Let  $x_{ij}$ ,  $j = 1, 2, \dots, n$  be a random sample from population  $\pi_i$ ,  $i = 0, 1, \dots, k$ . The sample space is denoted by  $\mathcal{X} = \{x = (x_{01}, \dots, x_{0n}, x_{11}, \dots, x_{1n}, \dots, x_{k1}, \dots, x_{kn}) | \mu_i < x_{ij} < \infty, j = 1, 2, \dots, n, i=0, 1, \dots, k\}$ .

**Definition 2.1.** A selection procedure  $\delta$  is isotonic if it selects  $\pi_i$  with parameter  $\mu_i$  and if  $\mu_i < \mu_j$ , then it also selects  $\pi_j$ . We will restrict our attention to isotonic selection procedure  $\delta$  which satisfy the  $P^*$ -condition:

$$\inf_{\mu \in \Omega} P_{\mu}(CS|\delta) \geq P^*,$$

where  $P^*$  is a pre-assigned value, and a correct selection (CS) means the selection of any subset which contains all good populations.

**Definition 2.2.** A poset  $(S, \leq)$  denotes a non-empty set  $S$  with a binary partial order  $\leq$  defined on it.

**Definition 2.3.** A real-valued function  $f$  defined on a poset  $(S, \leq)$  is called isotonic if  $f$  preserves the order on  $S$ , i.e.  $x \leq y$ , implies  $f(x) \leq f(y)$ .

**Definition 2.4.** Let  $g$  be a real-valued function and let  $W$  be a positive-valued function, both defined on a poset  $(S, \leq)$ . An isotonic function  $g^*$  on  $S$  is called an isotonic regression of  $g$  with weight  $W$  if  $\sum_{x \in S} [g(x) - g^*(x)]^2 W(x)$  attains its minimum values over set of all isotonic functions on  $S$ .

It is well-known (see Barlow, Bartholomew, Bremner and Brunk (1972)) that there exists one and only one isotonic regression of a given  $g$  with a given weight  $W$  defined on  $S$ .

Let  $Y_i = \min_{1 \leq j \leq n} X_{ij}$ , where  $X_{ij} \sim E(\mu_i, \theta)$ ,  $j = 1, 2, \dots, n$ ,  $i = 0, 1, \dots, k$ .

Let  $S = \{\mu_1, \dots, \mu_k \mid \mu_1 \leq \dots \leq \mu_k\}$ . Consider the functions  $g(\mu_i) = Y_i$  and  $W(\mu_i) = n/\theta = w_i$ ,  $i = 1, \dots, k$ . Then by the maximin formula, the isotonic regression of  $g$  with weight  $W$  is  $g^*$ , where

$$g^*(\mu_i) = \max_{1 \leq s \leq i} \min_{s \leq t \leq k} \left\{ \frac{Y_s + \dots + Y_t}{t - s + 1} \right\}.$$

The isotonic estimator of  $\mu_i$  is denoted by  $\hat{X}_{i:k}$ ,  $i = 1, 2, \dots, k$ , where

$$(2.2) \quad \hat{X}_{i:k} = \max_{1 \leq s \leq i} \hat{X}_{s:k}$$

and

$$(2.3) \quad \hat{X}_{s:k} = \min \left\{ Y_s, \frac{Y_s + Y_{s+1}}{2}, \dots, \frac{Y_s + \dots + Y_k}{k - s + 1} \right\}.$$

It is known that the isotonic estimators  $\hat{X}_{i:k}$ ,  $i = 1, \dots, k$  are also the maximum likelihood estimators of  $\mu_i$ ,  $i = 1, 2, \dots, k$ , for the two-parameter exponential distributions.



### 3. Isotonic Selection Procedures

#### 3.1. $\mu_0$ and $\theta$ are known

Let us define

$$\Omega_i = \{\mu \in \Omega \mid \mu_{k-1} < \mu_0 \leq \mu_{k-i+1}\}, \quad i = 1, 2, \dots, k-1,$$

$$\Omega_k = \{\mu \in \Omega \mid \mu_0 \leq \mu_1\},$$

and

$$\Omega_0 = \{\mu \in \Omega \mid \mu_k < \mu_0\}.$$

Then  $\Omega_i$  are disjoint and  $\Omega = \bigcup_{i=0}^k \Omega_i$ . Furthermore,

$$\inf_{\mu \in \Omega} P_\mu(CS|\delta) = \min_{1 \leq i \leq k} \inf_{\mu \in \Omega_i} P_\mu(CS|\delta), \quad \text{for any } \delta,$$

and

$$\inf_{\mu \in \Omega} P_\mu(CS|\delta) \geq P^* \text{ iff } \inf_{\mu \in \Omega_i} P_\mu(CS|\delta) \geq P^*, \quad i = 1, 2, \dots, k.$$

If  $\mu_0$  is known, no samples are drawn from  $\pi_0$  and  $\underline{X} = (X_{11}, \dots, X_{1n}, \dots, X_{k1}, \dots, X_{kn})$ . We propose a selection procedure  $\delta_1^{(1)}$  as follows:

$$(3.1) \quad \delta_1^{(1)}(\underline{X}) = a_{\underline{c}(\underline{X})}, \quad \text{where } \underline{c}(\underline{X}) = \min\{i \mid \hat{X}_{i:k} \geq \mu_0 + d_{i:k}^{(1)} \frac{\theta}{n}\},$$

here  $\hat{X}_{i:k}$  is defined by (2.2) and  $d_{i:k}^{(1)}$ ,  $i = 1, 2, \dots, k$  are determined to satisfy the  $P^*$ -condition.

**Lemma 3.1.** For any  $\mu \in \Omega_i$ ,  $1 \leq i \leq k$ ,  $P_\mu(CS|\delta_1^{(1)})$  is increasing in  $\mu_j$ ,  $1 \leq j \leq k$ .

**Proof.** If  $\mu \in \Omega_i$ , then

$$\begin{aligned} P_\mu(CS|\delta_1^{(1)}) &= P_\mu\left\{\bigcup_{j=1}^{k-i+1} \{\hat{X}_{j:k} \geq \mu_0 + d_{j:k}^{(1)} \frac{\theta}{n}\}\right\} \\ &= P_\mu\left\{\bigcup_{j=1}^{k-i+1} \bigcup_{r=1}^j \{\hat{X}_{r:k} \geq \mu_0 + d_{j:k}^{(1)} \frac{\theta}{n}\}\right\} \\ &= E_\mu(I_A(\underline{X})). \end{aligned}$$

where  $A = \bigcup_{j=1}^{k-i+1} \bigcup_{r=1}^j \{\hat{X}_{r:k} \geq \mu_0 + d_{j:k}^{(1)} \frac{\theta}{n}\}$ .

Since  $I_A(\underline{x})$  is increasing in  $(x_{j1}, \dots, x_{jn})$ ,  $1 \leq j \leq k$ , and the distribution of  $X_{ij}$  has stochastically increasing property, hence  $E_{\underline{\mu}}\{I_A(\underline{X})\}$  is increasing in  $\mu_j$ ,  $1 \leq j \leq k$ . This completes the proof of the lemma.

It follows from Lemma 3.1 that  $\inf_{\underline{\mu} \in \Omega_i} P_{\underline{\mu}}(CS|\delta_1^{(1)}) = P_{\underline{\mu}^*}(CS|\delta_1^{(1)})$ , where  $\underline{\mu}^* = (\mu_0, -\infty, \dots, -\infty, \underbrace{\mu_0, \dots, \mu_0}_{i \text{ terms}})$ , and

$$\begin{aligned} P_{\underline{\mu}^*}(CS|\delta_1^{(1)}) &= P_{\underline{\mu}^*}\{\hat{X}_{k-i+1:k} \geq \mu_0 + d_{k-i+1:k}^{(1)} \frac{\theta}{n}\} \\ &= P\{\hat{Z}_{k-i+1:k} \geq d_{k-i+1:k}^{(1)}\}, \end{aligned}$$

where  $Z_1, \dots, Z_k$  are i.i.d.  $E(0,1)$ .

Now  $\hat{Z}_{k-i+1:k}$  has the same distribution as  $\hat{Z}_{1:i}$ . If we let

$$(3.2) \quad V_i = \hat{Z}_{1:i} = \min_{1 \leq r \leq i} \left\{ \frac{1}{r} \sum_{j=1}^r Z_j \right\},$$

then we have

$$(3.3) \quad \inf_{\underline{\mu} \in \Omega_i} P_{\underline{\mu}}(CS|\delta_1^{(1)}) = P(V_i \geq d_{k-i+1:k}^{(1)}), \quad i = 1, 2, \dots, k,$$

and the following theorem follows.

**Theorem 3.2.** For given  $P^*(0 < P^* < 1)$ , if  $d_{k-i+1:k}^{(1)}$  is the solution to the equation  $P(V_i \geq x) = P^*$ , where  $V_i$  is defined by (3.2). Then  $\delta_1^{(1)}$  defined by (3.1) satisfies the  $P^*$ -condition.

Remarks:

- (1) If  $x \leq 0$ , then  $P(V_i \geq x) = 1$ , hence we restrict our attention to  $d_{i:k}^{(1)} > 0$ ,  $i = 1, 2, \dots, k$ .

- (2) It is clear that  $d_{k-i+1:k}^{(1)} = d_{1:i}^{(1)}$  for all  $1 \leq i \leq k$ . Furthermore,  $V_i \geq V_{i+1}$  implies  $d_{1:i}^{(1)}$  is decreasing in  $i$ .

In order to find the  $d_{i:k}^{(1)}$ 's we need to find the joint distribution of  $Z_1, Z_1+Z_2, \dots$ , and  $Z_1 + \dots + Z_i$ ,  $1 \leq i \leq k$ . Theorem 3.3 gives an explicit form to find  $d_{i:k}^{(1)}$ 's.

**Theorem 3.3.** For  $x > 0$ ,  $P(V_i \geq x) = e^{-ix} \sum_{j=1}^i b_j x^{j-1}$ ,  $1 \leq i \leq k$ , where

$$(3.4) \quad b_j = i^{(j-2)}(i-j+1)/(j-1)!.$$

**Proof.** Consider the transformation  $U_1 = Z_1$ ,  $U_2 = Z_1+Z_2, \dots$ ,  $U_i = Z_1 + \dots + Z_i$ , then  $U_1, \dots, U_i$  have joint pdf  $e^{-u_i}$ ,  $0 < u_1 < u_2 < \dots < u_i < \infty$ . Hence

$$\begin{aligned} P(V_i \geq x) &= \int_x^\infty e^{-u_i} \left( \frac{u_i^{i-1}}{(i-1)!} - \frac{u_i^{i-2}}{(i-2)!} x \right) du_i \\ &= \frac{(ix)^{i-1}}{(i-1)!} e^{-ix} - (x-1) \int_x^\infty e^{-u_i} \frac{u_i^{i-2}}{(i-2)!} du_i \\ &= e^{-ix} \sum_{j=1}^i b_j x^{j-1}, \end{aligned}$$

where  $b_j$  is defined by (3.4).

From Theorem 3.3, for  $1 \leq i \leq k$ ,  $d_{k-i+1:k}^{(1)}$  is the solution to the equation

$$(3.5) \quad e^{-ix} \sum_{j=1}^i b_j x^{j-1} = P^*, \text{ where } b_j \text{ is defined by (3.4).}$$

The values of  $d_{1:i}^{(1)} (= d_{k-i+1:k}^{(1)})$ , for  $k = 1(1)20$ , and  $P^* = 0.800(0.025) 0.975$  and  $0.990$  are tabulated in Table I.

### 3.2. $\mu_0$ known, $\theta$ unknown

If the common value of  $\theta$  is unknown, let  $\hat{\theta} = \sum_{i=1}^k \sum_{j=1}^n (X_{ij} - Y_i)/v$ , where  $v = k(n-1)$ . Then  $2v\hat{\theta}/\theta$  is distributed as chi-square with  $2v$  degrees of

freedom and is independent of  $Y_1, \dots, Y_k$  (see Epstein and Sobel (1954)). We propose a selection procedure  $\delta_1^{(2)}$  by

$$(3.6) \quad \delta_1^{(2)}(\underline{x}) = a_c(\underline{x}), \text{ where } c(\underline{x}) = \min\{i | \bar{x}_{i:k} \geq \nu_0 + d_{1:k}^{(2)} \frac{2v\hat{\theta}}{n}\}.$$

Analogous to the proof of Theorem 3.2, we have the following result.

**Theorem 3.4.** For given  $P^*(0 < P^* < 1)$ , if  $d_{k-1+1:k}^{(2)}$  is the solution to the equation  $P(V_1 \geq \frac{2v\hat{\theta}}{\theta} x) = P^*$ , where  $V_1$  is defined by (3.2) and  $2v\hat{\theta}/\theta \sim \chi_{2v}^2$  are independent, then  $\delta_1^{(2)}$  defined by (3.6) satisfies the  $P^*$ -condition.

$$(3.7) \quad \text{Theorem 3.5. For } x > 0, P(V_1 \geq \frac{2v\hat{\theta}}{\theta} x) = \sum_{j=1}^{\infty} b_j (2x)^{j-1} \frac{\Gamma(v+j-1)}{\Gamma(v)(1+2ix)^{v+j-1}},$$

where  $b_j$  is defined by (3.4).

**Proof.** The proof is straightforward.

#### Remarks

(1)  $d_{k-1+1:k}^{(2)}$  depend on  $v = k(n-1)$  and  $d_{k-1+1:k}^{(2)} \neq d_{1:1}^{(2)}$ .

(2)  $d_{k-1+1:k}^{(2)}$  is the solution to the equation

$$(3.7) \quad \sum_{j=1}^{\infty} b_j (2x)^{j-1} \frac{\Gamma(v+j-1)}{\Gamma(v)(1+2ix)^{v+j-1}} = P^*, x > 0, \text{ where}$$

$b_j$  is defined by (3.4).

The values of  $d_{k-1+1:k}^{(2)}$  for  $k = 2(1)6$ ,  $P^* = 0.900(0.025)0.975$  and  $0.990$ , with common sample size  $n = 5(5)20$  are tabulated in Table II.

#### 3.2. $\nu_0$ unknown, $\theta$ known

In the case where  $\nu_0$  is unknown, we take additional observations  $X_{0j}$ ,

$j = 1, 2, \dots, n$  from  $\nu_0$  and denote  $\underline{x}$  by  $(X_{01}, \dots, X_{0n}, X_{11}, \dots, X_{1n}, \dots, X_{k1}, \dots, X_{kn})$ .

Let  $\nu_0 = \nu$ ,  $\nu = 0, 1, 2, \dots, k-1$ . Let  $\bar{x}_{i:k}$  be defined as in (2.2). We propose a

selection procedure  $\delta_1^{(3)}$  by

$$(3.8) \quad \delta_1^{(3)}(\underline{x}) = a_{\varepsilon(\underline{x})}, \text{ where } \varepsilon(\underline{x}) = \min\{i | \hat{X}_{i:k} \geq Y_0 - d_{i:k}^{(3)} \frac{\theta}{n}\}.$$

**Theorem 3.6.** For given  $P^*$  ( $0 < P^* < 1$ ), if  $d_{k-i+1:k}^{(3)}$  is the solution to the equation

$$(3.9) \quad \sum_{j=1}^i b_j e^{-x} \int_{-x}^{\infty} z^{j-1} e^{-(i+1)z} dz = P^*, \quad x \leq 0,$$

or the equation

$$(3.10) \quad 1 - e^{-x} \left\{ 1 - \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j} \right\} = P^*, \quad x > 0,$$

where  $1 \leq i \leq k$  and  $b_j$  is defined by (3.4). Then  $\delta_1^{(3)}$  defined by (3.8) satisfies the  $P^*$ -condition.

**Proof.** If  $\underline{\mu} \in \Omega_1$ ,  $P_{\underline{\mu}}(CS|\delta_1^{(3)})$  is increasing in  $\mu_j$ ,  $1 \leq j \leq k$  and is decreasing in  $\mu_0$ . Hence

$$\inf_{\underline{\mu} \in \Omega_1} P_{\underline{\mu}}(CS|\delta_1^{(3)}) = \inf_{\underline{\mu}^*} P_{\underline{\mu}^*}(CS|\delta_1^{(3)}),$$

where  $\underline{\mu}^* = (\mu_0, \underbrace{-\infty, \dots, -\infty}_{i \text{ terms}}, \mu_0, \dots, \mu_0)$ ,

and is independent of  $\mu_0$ . Therefore

$$\begin{aligned} \inf_{\underline{\mu} \in \Omega_1} P_{\underline{\mu}}(CS|\delta_1^{(3)}) &= P\{V_1 \geq Z_0 - d_{k-i+1:k}^{(3)}\} \\ &= \begin{cases} \sum_{j=1}^i b_j e^{-d_{k-i+1:k}^{(3)}} \int_{-d_{k-i+1:k}^{(3)}}^{\infty} z^{j-1} e^{-(i+1)z} dz, & \text{if } d_{k-i+1:k}^{(3)} \leq 0 \\ 1 - e^{-d_{k-i+1:k}^{(3)}} \left\{ 1 - \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j} \right\}, & \text{if } d_{k-i+1:k}^{(3)} > 0. \end{cases} \end{aligned}$$

Remarks:

(1) If  $d_{k-i+1:k}^{(3)} \leq 0$ , then  $P(V_1 \geq Z_0 - d_{k-i+1:k}^{(3)}) \leq P(V_1 \geq Z_0) \leq P(V_1 \geq Z_0) = 1/2$ .

Hence, for  $P^* > 1/2$ , there is no solution in the case when  $d_{k-i+1:k}^{(3)} \leq 0$ . We should restrict attention to  $d_{k-i+1:k}^{(3)} > 0$  and use the equation (3.10) or

$$d_{k-i+1:k}^{(3)} = -\ln((1-P^*)/(1 - \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j})).$$

The values of  $d_{1:k}^{(3)}$ , for  $k = 1(1)20$ , and  $P^* = 0.800(0.025)0.975$  and  $0.990$  are tabulated in Table III.

(2)  $d_{k-i+1:k}^{(3)} = d_{1:i}^{(3)}$  is increasing in  $i$ ,  $1 \leq i \leq k$ .

(3) If  $P^* > 1/2$ , then  $0 < (1-P^*)/(1 - \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j}) < 1$  and hence  $d_{k-i+1:k}^{(3)} > 0$ .

$$(4) \int_x^\infty z^{j-1} e^{-(i+1)z} dz = \frac{\Gamma(j)}{(i+1)^j} \sum_{\ell=0}^{j-1} \frac{((i+1)x)^\ell}{\ell!} e^{-(i+1)x}, \quad x > 0.$$

If  $d_{k-i+1:k}^{(3)} \leq 0$ , then  $-d_{k-i+1:k}^{(3)}$  is the solution to the equation

$$(3.11) \quad \left( \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j} \sum_{\ell=0}^{j-1} \frac{((i+1)x)^\ell}{\ell!} \right) e^{-ix} = P^*, \quad x \geq 0.$$

### 3.4. $\mu_0$ unknown, $\theta$ unknown

We define a selection procedure  $\delta_1^{(4)}$  by

$$(3.12) \quad \delta_1^{(4)}(\underline{X}) = a_{\epsilon}(\underline{X}), \text{ where } \epsilon(\underline{X}) = \min\{i | \hat{X}_{i:k} \geq Y_0 - d_{i:k}^{(4)} \frac{2v_1 \hat{\theta}_1}{n}\},$$

$$\text{where } v_1 = (k+1)(n-1), \hat{\theta}_1 = \sum_{i=0}^k \sum_{j=1}^n (X_{ij} - Y_i) / v_1.$$

**Theorem 3.7.** For given  $P^*(0 < P^* < 1)$ , if  $d_{k-i+1:k}^{(4)}$  is the solution to the equation

$$(3.13) \quad \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j} \sum_{\ell=0}^{j-1} \frac{(i+1)^\ell (-2x)^\ell \Gamma(v_1 + \ell)}{\ell! \Gamma(v_1) (1-2ix)^{v_1 + \ell}} = P^*, \quad x \leq 0$$

or the equation

$$(3.14) \quad 1 - \left(1 - \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j}\right) (1+2x)^{-v_1} = P^*, \quad x > 0,$$

where  $b_j$  is defined by (3.4). Then  $\delta_1^{(4)}$  defined by (3.12) satisfies the  $P^*$ -condition.

$$\begin{aligned} \text{Proof.} \quad \inf_{\mu \in \Omega_i} P_\mu(CS | \delta_1^{(4)}) &= P(V_1 \geq Z_0 - d_{k-i+1:k}^{(4)} \frac{2v_1 \hat{\theta}_1}{\theta}) \\ &= \begin{cases} \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j} \sum_{\ell=0}^{j-1} \frac{(i+1)^\ell (-2d_{k-i+1:k}^{(4)})^\ell \Gamma(v_1 + \ell)}{\ell! \Gamma(v_1) (1-2id_{k-i+1:k}^{(4)})^{v_1 + \ell}}, & \text{if } d_{k-i+1:k}^{(4)} \leq 0 \\ 1 - \left(1 - \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j}\right) (1+2d_{k-i+1:k}^{(4)})^{-v_1}, & \text{if } d_{k-i+1:k}^{(4)} > 0. \end{cases} \end{aligned}$$

Remark: If  $d_{k-i+1:k}^{(4)} \leq 0$ , then  $\inf_{\mu \in \Omega_i} P_\mu(CS | \delta_1^{(4)}) \leq 1/2$ . Hence, if  $P^* > 1/2$ ,  $d_{k-i+1:k}^{(4)}$  is the solution to the equation (3.14) or

$$d_{k-i+1:k}^{(4)} = \{[(1-P^*) / (1 - \sum_{j=1}^i b_j \frac{\Gamma(j)}{(i+1)^j})]^{-1/v_1} - 1\} / 2.$$

The values of  $d_{i:k}^{(4)}$  for  $k = 2(1)6$ ,  $P^* = 0.900(0.025)0.975$  and  $0.990$ , with common sample size  $n = 5(5)20$  are tabulated in Table IV.

#### 4. Some Other Proposed Selection Procedures

##### 4.1. $\mu_0$ and $\theta$ known

(1) In Section 3.1, if we take  $d = d_{1:k}^{(1)}$  and define a selection procedure  $\delta_2^{(1)}$  by

$$(4.1) \quad \delta_2^{(1)}: \text{ Select } \pi_i \text{ iff } \hat{X}_{i:k} \geq \mu_0 + d \frac{\theta}{n}, \quad i = 1, 2, \dots, k.$$

Since  $d_{1:k}^{(1)} = \min_{1 \leq i \leq k} d_{1:i}^{(1)}$ , it is easy to see that  $\inf_{\mu \in \Omega} P_{\mu}(CS|\delta_2^{(1)}) \geq P^*$ .

Furthermore,  $\hat{X}_{i:k} \geq \hat{X}_{j:k}$  for  $i > j$  implies  $\delta_2^{(1)}$  is an isotonic selection procedure.

(2) Let  $\bar{X}_j = \max\{Y_1, \dots, Y_j\}$ ,  $1 \leq j \leq k$  and define a selection procedure  $\delta_3^{(1)}$  by

$$(4.2) \quad \delta_3^{(1)}(\underline{X}) = a_{\varepsilon(\underline{X})}, \text{ where } \varepsilon(\underline{X}) = \min\{i | \bar{X}_i \geq \mu_0 + d_i \frac{\theta}{n}\}.$$

Then, for any  $i$ ,  $1 \leq i \leq k$

$$\inf_{\mu \in \Omega_i} P_{\mu}(CS|\delta_3^{(1)}) = P(Z_{k-i+1} \geq d_{k-i+1}) = e^{-d_{k-i+1}}.$$

If  $d_{k-i+1} = -\ln P^*$  for all  $i$ , then  $\delta_3^{(1)}$  satisfies the  $P^*$ -condition.

Remark:  $\delta_3^{(1)}$  is equivalent to:

$$\delta_3^{(1)}: \text{ Select } \pi_i \text{ iff } \bar{X}_i \geq \mu_0 - \ln P^* \frac{\theta}{n}, \quad i = 1, 2, \dots, k.$$

(3) Gupta and Sobel (1958) proposed a selection procedure without assuming any ordering prior. If we define a similar selection procedure  $\delta_4^{(1)}$  by

$$(4.3) \quad \delta_4^{(1)}: \text{ Select } \pi_i \text{ iff } Y_i \geq \mu_0 + d \frac{\theta}{n}, \quad i = 1, 2, \dots, k,$$

then

$$\inf_{\mu \in \Omega_i} P_{\mu}(CS|\delta_4^{(1)}) = e^{-id}, \quad \text{if } d > 0.$$

Hence  $\inf_{\mu \in \Omega} P_{\mu}(CS|\delta_4^{(1)}) = e^{-kd}$  and  $d = -\frac{1}{k} \ln P^*$ .

Note that the selection procedure  $\delta_4^{(1)}$  is not isotonic.

#### 4.2. $\mu_0$ known, $\theta$ unknown

(1) Similar to Section 3.2, we can define a selection procedure  $\delta_2^{(2)}$  by



(4.4)  $\delta_2^{(2)}$ : Select  $\pi_i$  iff  $\hat{X}_{i:k} \geq \mu_0 + d \frac{2v\hat{\theta}}{n}$ ,  $i = 1, 2, \dots, k$ ,

where  $d = d_{1:k}^{(2)}$ .

(2) We define a selection procedure  $\delta_3^{(2)}$  by

(4.5)  $\delta_3^{(2)}$ : Select  $\pi_i$  iff  $\tilde{X}_i \geq \mu_0 + d \frac{2v\hat{\theta}}{n}$ ,  $i = 1, 2, \dots, k$ ,

where  $d = ((P^*)^{-1/v} - 1)/2$ .

(3) We define a selection procedure  $\delta_4^{(2)}$  by

(4.6)  $\delta_4^{(2)}$ : Select  $\pi_i$  iff  $Y_i \geq \mu_0 + d \frac{2v\hat{\theta}}{n}$ ,  $i = 1, 2, \dots, k$ ,

where  $d = ((P^*)^{-1/v} - 1)/2k$ . Then  $\delta_i^{(2)}$ ,  $i = 2, 3, 4$  satisfy the  $P^*$ -condition.

#### 4.3. $\mu_0$ unknown, $\theta$ known

(1) If we define  $\delta_2^{(3)}$  by

(4.7)  $\delta_2^{(3)}$ : Select  $\pi_i$  iff  $\hat{X}_{i:k} \geq Y_0 - d \frac{\theta}{n}$ ,  $i = 1, 2, \dots, k$ ,

where  $d = d_{1:k}^{(3)}$ .

(2) If we define  $\delta_3^{(3)}$  by

(4.8)  $\delta_3^{(3)}$ : Select  $\pi_i$  iff  $\tilde{X}_i \geq Y_0 - d \frac{\theta}{n}$ ,  $i = 1, 2, \dots, k$ ,

where

$$d = \begin{cases} \ln 2P^* & , \text{ if } P^* \leq 1/2 \\ -\ln 2(1-P^*) & , \text{ if } P^* > 1/2. \end{cases}$$

(3) If we define  $\delta_4^{(3)}$  by

(4.9)  $\delta_4^{(3)}$ : Select  $\pi_i$  iff  $Y_i \geq Y_0 - d \frac{\theta}{n}$ ,  $i = 1, 2, \dots, k$ ,

where

$$d = \begin{cases} \frac{1}{k} \ln(k+1)P^*, & \text{if } P^* \leq 1/(k+1) \\ -\ln \frac{k+1}{k} (1-P^*), & \text{if } P^* > 1/(k+1). \end{cases}$$

Then  $\delta_i^{(3)}$ ,  $i = 2, 3, 4$  satisfy the  $P^*$ -condition.

#### 4.4. $\mu_0$ unknown, $\theta$ unknown

(1) We define  $\delta_2^{(4)}$  by

$$(4.10) \quad \delta_2^{(4)}: \text{ Select } \pi_i \text{ iff } \hat{x}_{i:k} \geq Y_0 - d \frac{2v_1 \hat{\theta}_1}{n}, \quad i = 1, 2, \dots, k,$$

where  $d = d_{1:k}^{(4)}$ .

(2) We define  $\delta_3^{(4)}$  by

$$(4.11) \quad \delta_3^{(4)}: \text{ Select } \pi_i \text{ iff } \tilde{x}_i \geq Y_0 - d \frac{2v_1 \hat{\theta}_1}{n}, \quad i = 1, 2, \dots, k,$$

where

$$d = \begin{cases} \{1 - (2P^*)^{-1/v_1}\}/2, & \text{if } P^* \leq 1/2 \\ \{2(1-P^*)\}^{-1/v_1} - 1/2, & \text{if } P^* > 1/2. \end{cases}$$

(3) We define  $\delta_4^{(4)}$  by

$$(4.12) \quad \delta_4^{(4)}: \text{ Select } \pi_i \text{ iff } Y_i \geq Y_0 - d \frac{2v_1 \hat{\theta}_1}{n}, \quad i = 1, 2, \dots, k,$$

where

$$d = \begin{cases} \{1 - ((k+1)P^*)^{-1/v_1}\}/2k, & \text{if } P^* \leq 1/(k+1) \\ \{((\frac{k+1}{k})(1-P^*))^{-1/v_1} - 1\}/2, & \text{if } P^* > 1/(k+1). \end{cases}$$

Then  $\delta_i^{(4)}$ ,  $i = 2, 3, 4$  satisfy the  $P^*$ -condition.

### 5. Expected Number (Size) of Bad Populations in the Selected Subset

In this section, we assume that  $\mu_0$  and  $\theta$  are known. Let  $E(S'|\delta)$  denote the expected number of bad populations in the selected subset when the selection procedure  $\delta$  is used. For the procedure satisfying the  $P^*$ -condition, usually we want the procedure with small expected number of bad populations in the selected subset. For procedure  $\delta_1^{(1)}$  we have the following theorem:

**Theorem 5.1.** For any  $j$ ,  $0 \leq j \leq k$ ,  $\sup_{\mu \in \Omega_{k-j}} E_{\mu}(S'|\delta_1^{(1)})$   
 $= \sum_{r=1}^j P\{\bigcup_{h=1}^r \{Z_{h:j} \geq d_{h:k}^{(1)}\}\}$ , where  $\hat{Z}_{h:j}$  is defined as in (2.3) and  $Z_1, \dots, Z_k$  are i.i.d.  $E(0,1)$ .

**Proof.** For any  $j$ ,  $0 \leq j \leq k$ , if  $\mu \in \Omega_{k-j}$ , we have

$$\begin{aligned} E_{\mu}(S'|\delta_1^{(1)}) &= \sum_{r=1}^j P_{\mu}(\pi_r \text{ is selected}|\delta_1^{(1)}) \\ &= \sum_{r=1}^j P_{\mu}\{\bigcup_{h=1}^r \{\hat{X}_{h:k} \geq \mu_0 + d_{h:k}^{(1)} \frac{\theta}{n}\}\}. \end{aligned}$$

Using the property similar to Lemma 3.1, we have

$$\begin{aligned} \sup_{\mu \in \Omega_{k-j}} E_{\mu}(S'|\delta_1^{(1)}) &= \sum_{r=1}^j P_{\mu^{**}}\{\bigcup_{h=1}^r \{\hat{X}_{h:k} \geq \mu_0 + d_{h:k}^{(1)} \frac{\theta}{n}\}\}. \\ &= \sum_{r=1}^j P\{\bigcup_{h=1}^r \{\hat{Z}_{h:j} \geq d_{h:k}^{(1)}\}\}, \end{aligned}$$

where  $\mu^{**} = (\underbrace{\mu_0, \mu_0, \dots, \mu_0}_{j \text{ terms}}, \infty, \dots, \infty)$ .

$j$  terms

For procedure  $\delta_2^{(1)}$ , it is easy to show that

$$\sup_{\mu \in \Omega_{k-j}} E_{\mu}(S'|\delta_2^{(1)}) = \sum_{r=1}^j P\{\bigcup_{h=1}^r \{\hat{Z}_{h:j} \geq d_{1:k}^{(1)}\}\} \text{ and}$$

$$\sup_{\mu \in \Omega_{k-j}} E_{\mu}(S' | \delta_1^{(1)}) \leq \sup_{\mu \in \Omega_{k-j}} E_{\mu}(S' | \delta_2^{(1)}), \text{ for } 0 \leq j \leq k.$$

Hence  $\delta_1^{(1)}$  is uniformly better than  $\delta_2^{(1)}$ . Furthermore,

$$\sup_{\mu \in \Omega} E(S' | \delta_2^{(1)}) = \sup_{\mu \in \Omega_0} E_{\mu}(S' | \delta_2^{(1)}), \text{ since}$$

$$\begin{aligned} \sum_{r=1}^{j+1} P\left\{ \bigcup_{h=1}^r \{\hat{Z}_{h:j+1} \geq d\} \right\} &\geq \sum_{r=2}^{j+1} P\left\{ \bigcup_{h=2}^r \{\hat{Z}_{h:j+1} \geq d\} \right\} \\ &= \sum_{r=1}^j P\left\{ \bigcup_{h=1}^r \{\hat{Z}_{h:j} \geq d\} \right\}. \end{aligned}$$

For procedure  $\delta_3^{(1)}$ , we have the following theorem:

**Theorem 5.2.** For any  $j$ ,  $0 \leq j \leq k$ ,  $\sup_{\mu \in \Omega_{k-j}} E(S' | \delta_3^{(1)}) = j - q(1 - q^j)/p^*$  and

$$\sup_{\mu \in \Omega} E(S' | \delta_3^{(1)}) = k - q(1 - q^k)/p^*, \text{ where } q = 1 - p^*.$$

**Proof.**  $\sup_{\mu \in \Omega_{k-j}} E(S' | \delta_3^{(1)}) = \sup_{\mu \in \Omega_{k-j}} \sum_{r=1}^j P\left\{ \max_{1 \leq s \leq r} Y_s \geq \mu_0 + d \frac{\theta}{n} \right\}$

$$= \sum_{r=1}^j \{1 - P\{\max_{1 \leq s \leq r} Z_s < d\}\} = j - q(1 - q^j)/p^*,$$

and  $\sup_{\mu \in \Omega_{k-j}} E(S' | \delta_3^{(1)})$  is increasing in  $j$ .

In order to compare the procedures  $\delta_1^{(1)}$  and  $\delta_3^{(1)}$ , we need the following

lemma:

**Lemma 5.3.** For  $i = 1, \dots, k$ , let  $A_i = \{\hat{Z}_{i:k} \geq d_{i:k}^{(1)}\}$ , then

$$P\left\{ \bigcup_{i=1}^j A_i \cap A_{j+1} \right\} > P\left\{ \bigcup_{i=1}^j A_i \right\} p^* \text{ for all } j, 1 \leq j \leq k-1, k \geq 2.$$

Proof. If  $\hat{z}_{j+1:k} \geq d_{j+1:k}^{(1)}$  and  $\hat{z}_{i:j} \geq d_{i:k}^{(1)}$  for some  $i$ ,  $1 \leq i \leq j$ , then

$$\begin{aligned} \frac{z_i + \dots + z_k}{k-i+1} &= \frac{(j-i+1)(z_i + \dots + z_j)/(j-i+1) + (k-j)(z_{j+1} + \dots + z_k)/(k-j)}{k-i+1} \\ &\geq \frac{(j-i+1)d_{i:k}^{(1)} + (k-j)d_{j+1:k}^{(1)}}{k-i+1} = d_{i:k}^{(1)}, \text{ for } j+1 \leq k \leq k. \end{aligned}$$

Hence

$$\begin{aligned} P\left(\bigcup_{i=1}^j A_i \cap A_{j+1}\right) &= P\left(\bigcup_{i=1}^j \{\hat{z}_{i:j} \geq d_{i:k}^{(1)}\} \cap A_{j+1}\right) \\ &= P\left(\bigcup_{i=1}^j \{\hat{z}_{i:j} \geq d_{i:k}^{(1)}\}\right) P(A_{j+1}) \\ &> P\left(\bigcup_{i=1}^j A_i\right) P^*. \end{aligned}$$

Theorem 5.4. For all  $k \geq 2$ ,  $\sup_{\mu \in \Omega_0} E_{\mu}(S' | \delta_1^{(1)}) \leq \sup_{\mu \in \Omega_0} E_{\mu}(S' | \delta_3^{(1)})$ .

Proof. By Lemma 5.3 and the induction principle, we have

$$P\left(\bigcup_{i=1}^j A_i\right) \leq 1 - (1 - P^*)^j \text{ for all } j, 1 \leq j \leq k.$$

Hence

$$\begin{aligned} \sup_{\mu \in \Omega_0} E(S' | \delta_1^{(1)}) &= \sum_{r=1}^k P\left(\bigcup_{h=1}^r \{\hat{z}_{h:k} \geq d_{h:k}^{(1)}\}\right) \\ &\leq \sum_{r=1}^k \{1 - (1 - P^*)^r\} = \sup_{\mu \in \Omega_0} E(S' | \delta_3^{(1)}). \end{aligned}$$

Remark: Theorem 5.4 tells us that procedure  $\delta_1^{(1)}$  is better than  $\delta_3^{(1)}$  in the sense that in  $\Omega_0$  it tends to select smaller number of bad populations, however, procedure  $\delta_1$  is not uniformly better than  $\delta_3^{(1)}$ .

In order to compare the procedures  $\delta_3^{(1)}$  and  $\delta_4^{(1)}$ , we need the following lemma.

**Lemma 5.5.**  $k(1-(p^*)^{1/k}) + \ln p^*, \quad 0 < p^* < 1.$

**Proof.** Let  $f(k) = k(1-(p^*)^{1/k})$ , then

$$f'(k) = 1-(p^*)^{1/k} + \frac{1}{k} (\ln p^*) (p^*)^{1/k}.$$

$$f'(k) > 0 \quad \text{iff} \quad -\frac{1}{k} \ln p^* > \ln(1 - \frac{\ln p^*}{k}).$$

The result follows since  $-\frac{1}{k} \ln p^* > 0$  and  $\lim_{k \rightarrow \infty} k(1-(p^*)^{1/k}) = -\ln p^*.$

**Theorem 5.6.** If  $k \geq 2$  and  $p^* > 1/2$ , then  $\sup_{\mu \in \Omega} E(S' | \delta_3^{(1)}) < \sup_{\mu \in \Omega} E(S' | \delta_4^{(1)}).$

**Proof.** It is easy to show that  $\sup_{\mu \in \Omega_{k-j}} E_{\mu}(S' | \delta_4^{(1)}) = j(p^*)^{1/k}$  and hence

$$\sup_{\mu \in \Omega} E_{\mu}(S' | \delta_4^{(1)}) = k(p^*)^{1/k}.$$

$$\sup_{\mu \in \Omega} E_{\mu}(S' | \delta_3^{(1)}) < \sup_{\mu \in \Omega} E_{\mu}(S' | \delta_4^{(1)}) \quad \text{iff}$$

$$k(1-(p^*)^{1/k}) < (1-p^*)(1-(1-p^*)^k)/p^*.$$

If  $p^* > 1/2$ , then  $-\ln p^* < (1-p^*)(2-p^*)$ . By Lemma 5.5, we have

$$k(1-(p^*)^{1/k}) < -\ln p^*.$$

$$\sup_{\mu \in \Omega} E_{\mu}(S' | \delta_3^{(1)}) < \sup_{\mu \in \Omega} E_{\mu}(S' | \delta_4^{(1)}), \text{ since}$$

$$(1-p^*)(2-p^*) = (1-p^*)(1-(1-p^*)) / p^*$$

$$< (1-p^*)(1-(1-p^*)^k) / p^*.$$

**Remark:** Theorem 5.6 tells us that procedure  $\delta_3^{(1)}$  is uniformly better than procedure  $\delta_4^{(1)}$ .

Table I

Table of  $d_{1:k}^{(1)}$  values associated with procedure  $\delta_1^{(1)}$ .

$d_{1:k}^{(1)}$ k	$p^*$								
	0.990	0.975	0.950	0.925	0.900	0.875	0.850	0.825	0.800
1	.0100	.0253	.0512	.0779	.1053	.1335	.1625	.1923	.2231
2	-	.0250	.0500	.0752	.1006	.1261	.1520	.1781	.2046
3	-	-	-	.0750	.1000	.1252	.1504	.1757	.2012
4	-	-	-	-	-	.1250	.1501	.1752	.2004
5	-	-	-	-	-	-	.1500	.1750	.2001
6	-	.0249	-	-	-	-	-	-	.2000
7-20*	-	-	-	-	-	-	-	-	-

The "-" in Table I means that the value is the same as the preceding one in the same column.

\* For  $k = 7(1)20$ , values of  $d_{1:k}^{(1)}$  are the same for any given  $P^*$  in the above table.

Table II

Table of  $d_{i:k}^{(2)}$  values associated with procedure  $\delta_1^{(2)}$ .

		n = 5					n = 10				
$k \backslash i$		0.990	0.975	0.950	0.925	0.900	0.990	0.975	0.950	0.925	0.900
2	1	.0006	.0015	.0031	.0047	.0063	.0002	.0006	.0013	.0020	.0027
	2	-	-	.0032	.0048	.0066	-	.0007	.0014	.0021	.0029
3	1	.0004	.0010	.0020	.0031	.0041	.0001	.0004	.0009	.0013	.0018
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	.0021	.0032	.0044	-	-	-	.0014	.0019
4	1	.0003	.0007	.0015	.0023	.0031	.0001	.0003	.0006	.0010	.0013
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	4	-	-	.0016	.0024	.0033	-	-	.0007	-	.0014
5	1	.0002	.0006	.0012	.0018	.0025	.0001	.0002	.0005	.0008	.0011
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	4	-	-	-	-	-	-	-	-	-	-
	5	-	-	-	.0019	.0026	-	-	-	-	-
6	1	.0002	.0005	.0010	.0015	.0020	.0000	.0002	.0004	.0006	.0009
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	4	-	-	-	-	-	-	-	-	-	-
	5	-	-	-	-	-	-	-	-	-	-
	6	-	-	-	.0016	.0021	-	-	-	.0007	-

The "-" in Table II means that the value is the same as the preceding one in the same column.



Table II (continued)

Table of  $d_{i:k}^{(2)}$  values associated with procedure  $\delta_i^{(2)}$ .

		n = 15					n = 20				
$p \backslash k$		0.990	0.975	0.950	0.925	0.900	0.990	0.975	0.950	0.925	0.900
2	1	.0001	.0004	.0008	.0013	.0017	.0001	.0003	.0006	.0009	.0013
	2	-	-	.0009	-	.0018	-	-	-	.0010	-
3	1	.0001	.0002	.0005	.0008	.0011	.0000	.0002	.0004	.0006	.0008
	2	-	-	-	-	-	-	-	-	-	-
	3	-	.0003	.0006	.0009	.0012	-	-	-	-	.0009
4	1	.0000	.0002	.0004	.0006	.0008	.0000	.0001	.0003	.0004	.0006
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	4	-	-	-	-	.0009	-	-	-	.0005	-
5	1	.0000	.0001	.0003	.0005	.0007	.0000	.0001	.0002	.0003	.0005
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	4	-	-	-	-	-	-	-	-	-	-
	5	-	-	-	-	-	-	-	-	.0004	-
6	1	.0000	.0001	.0002	.0004	.0005	.0000	.0001	.0002	.0003	.0004
	2	-	-	-	-	-	-	-	-	-	-
	3	-	-	-	-	-	-	-	-	-	-
	4	-	-	-	-	-	-	-	-	-	-
	5	-	-	-	-	-	-	-	-	-	-
	6	-	-	.0003	-	.0006	-	-	-	-	-

The "-" in Table II means that the value is the same as the preceding one in the same column.

Table III

Table of  $d_{1:k}^{(3)}$  values associated with procedure  $\delta_1^{(3)}$ .

$d_{1:k}^{(3)}$		$p^*$								
$k$		0.990	0.975	0.950	0.925	0.900	0.875	0.850	0.825	0.800
1		3.9121	2.9958	2.3026	1.8972	1.6095	1.3863	1.2040	1.0499	0.9163
2		4.0574	3.1011	2.4080	2.0025	1.7148	1.4917	1.3094	1.1552	1.0217
3		4.0783	3.1410	2.4478	2.0424	1.7547	1.5315	1.3492	1.1951	1.0615
4		4.0913	3.1620	2.4688	2.0634	1.7757	1.5525	1.3702	1.2161	1.0825
5		4.0913	3.1750	2.4818	2.0764	1.7887	1.5655	1.3832	1.2291	1.0955
6		4.1001	3.1838	2.4907	2.0852	1.7975	1.5744	1.3920	1.2379	1.1044
7		4.1065	3.1902	2.4971	2.0916	1.8039	1.5808	1.3984	1.2443	1.1108
8		4.1112	3.1951	2.5019	2.0964	1.8088	1.5856	1.4033	1.2491	1.1156
9		4.1151	3.1989	2.5057	2.1002	1.8126	1.5894	1.4071	1.2529	1.1194
10		4.1182	3.2019	2.5088	2.1033	1.8156	1.5925	1.4102	1.2560	1.1225
11		4.1207	3.2044	2.5113	2.1058	1.8181	1.5950	1.4127	1.2585	1.1250
12		4.1228	3.2065	2.5134	2.1079	1.8202	1.5971	1.4148	1.2606	1.1271
13		4.1246	3.2083	2.5152	2.1097	1.8220	1.5989	1.4166	1.2624	1.1289
14		4.1262	3.2099	2.5167	2.1112	1.8236	1.6004	1.4181	1.2640	1.1304
15		4.1275	3.2112	2.5180	2.1126	1.8249	1.6018	1.4194	1.2653	1.1318
16		4.1287	3.2124	2.5192	2.1138	1.8261	1.6029	1.4206	1.2665	1.1329
17		4.1297	3.2134	2.5203	2.1148	1.8271	1.6040	1.4216	1.2675	1.1340
18		4.1306	3.2143	2.5212	2.1157	1.8280	1.6049	1.4226	1.2684	1.1349
19		4.1314	3.2152	2.5220	2.1165	1.8289	1.6057	1.4234	1.2692	1.1357
20		4.1322	3.2159	2.5228	2.1173	1.8296	1.6065	1.4241	1.2700	1.1365

Table IV

Table of  $d_{i:k}^{(4)}$  values associated with procedure  $\delta_1^{(4)}$ .

		n = 5					n = 10				
k	p*	0.990	0.975	0.950	0.925	0.900	0.990	0.975	0.950	0.925	0.900
	1										
2	1	.1989	.1475	.1112	.0909	.0769	.0803	.0609	.0467	.0385	.0328
	2	.1928	.1418	.1058	.0857	.0718	.0780	.0587	.0446	.0364	.0308
3	1	.1444	.1086	.0827	.0681	.0580	.0597	.0456	.0352	.0292	.0250
	2	.1428	.1070	.0813	.0667	.0566	.0591	.0450	.0346	.0287	.0244
	3	.1385	.1030	.0774	.0630	.0530	.0574	.0434	.0331	.0271	.0229
4	1	.1131	.0857	.0657	.0544	.0465	.0475	.0364	.0282	.0235	.0202
	2	.1125	.0851	.0651	.0538	.0459	.0472	.0362	.0280	.0233	.0199
	3	.1113	.0839	.0640	.0527	.0448	.0467	.0357	.0275	.0228	.0195
	4	.1081	.0808	.0611	.0498	.0419	.0455	.0345	.0263	.0216	.0183
5	1	.0930	.0708	.0545	.0452	.0387	.0394	.0303	.0236	.0196	.0169
	2	.0927	.0705	.0542	.0449	.0384	.0393	.0302	.0234	.0195	.0168
	3	.0921	.0700	.0537	.0445	.0380	.0391	.0300	.0232	.0193	.0166
	4	.0912	.0690	.0528	.0436	.0371	.0387	.0296	.0229	.0189	.0162
	5	.0886	.0665	.0504	.0412	.0347	.0376	.0286	.0218	.0179	.0152
6	1	.0789	.0603	.0466	.0387	.0332	.0337	.0260	.0202	.0169	.0145
	2	.0787	.0601	.0464	.0385	.0330	.0336	.0259	.0201	.0168	.0144
	3	.0784	.0598	.0461	.0383	.0328	.0335	.0258	.0200	.0167	.0143
	4	.0780	.0594	.0457	.0379	.0324	.0333	.0256	.0199	.0165	.0142
	5	.0772	.0586	.0450	.0371	.0316	.0330	.0253	.0195	.0162	.0138
	6	.0750	.0565	.0429	.0351	.0296	.0321	.0244	.0187	.0153	.0130

Table IV (continued)

Table of  $d_{i:k}^{(4)}$  values associated with procedure  $\delta_1^{(4)}$ .

		n = 15					n = 20				
$\backslash p^*$	$k$	0.990	0.975	0.950	0.925	0.900	0.990	0.975	0.950	0.925	0.900
2	1	.0502	.0384	.0296	.0245	.0209	.0366	.0280	.0216	.0179	.0153
	2	.0489	.0370	.0282	.0232	.0196	.0356	.0270	.0207	.0170	.0144
3	1	.0376	.0289	.0224	.0186	.0160	.0275	.0211	.0164	.0137	.0117
	2	.0372	.0285	.0220	.0183	.0156	.0272	.0209	.0161	.0134	.0115
	3	.0362	.0275	.0210	.0173	.0146	.0265	.0202	.0154	.0127	.0108
4	1	.0300	.0232	.0180	.0150	.0129	.0220	.0170	.0132	.0110	.0095
	2	.0299	.0230	.0178	.0149	.0127	.0219	.0169	.0131	.0109	.0094
	3	.0296	.0227	.0175	.0146	.0124	.0216	.0166	.0129	.0107	.0092
	4	.0288	.0219	.0168	.0138	.0117	.0211	.0161	.0123	.0101	.0086
5	1	.0250	.0193	.0150	.0126	.0108	.0183	.0142	.0111	.0092	.0080
	2	.0249	.0192	.0150	.0125	.0107	.0183	.0141	.0110	.0092	.0079
	3	.0248	.0191	.0148	.0124	.0106	.0182	.0140	.0109	.0091	.0078
	4	.0245	.0189	.0146	.0121	.0104	.0180	.0138	.0107	.0089	.0076
	5	.0239	.0182	.0139	.0115	.0097	.0175	.0134	.0103	.0084	.0072
6	1	.0214	.0166	.0129	.0108	.0093	.0157	.0122	.0095	.0080	.0069
	2	.0214	.0165	.0129	.0108	.0093	.0157	.0121	.0095	.0079	.0068
	3	.0213	.0164	.0128	.0107	.0092	.0156	.0121	.0094	.0079	.0068
	4	.0212	.0163	.0127	.0106	.0091	.0155	.0120	.0093	.0078	.0067
	5	.0210	.0161	.0125	.0104	.0089	.0154	.0118	.0092	.0076	.0065
	6	.0204	.0156	.0119	.0098	.0083	.0150	.0114	.0088	.0072	.0061

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
Technical Report #83-46	AD-413545	
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED
ISOTONIC PROCEDURES FOR SELECTING POPULATIONS BETTER THAN A STANDARD: TWO-PARAMETER EXPONENTIAL DISTRIBUTIONS		Technical
7. AUTHOR(s)		8. CONTRACT OR GRANT NUMBER(s)
Shanti S. Gupta and Lii-Yuh Leu		N00014-75-C-0455
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Purdue University Department of Statistics West Lafayette, IN 47907		
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE
Office of Naval Research Washington, DC		October 1983
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES
		24
		15. SECURITY CLASS. (of this report)
		Unclassified
		15a. DECLASSIFICATION, DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)		
Approved for public release, distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
Isotonic procedures, selection procedures, standard, negative exponential, guarantee time, subset selection, simple ordering prior.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
The problem of selecting populations, from two-parameter exponential populations, which are better than a standard under an ordering prior is investigated. If the negative exponential distribution is the model for lifetime, then the problem is to select all those populations for which the guarantee lifetimes are larger than that of a standard. Comparisons of these procedures based on the expected number of bad populations in the selected subset is investigated. Tables of associated constants for the proposed procedures are given in Table I through Table IV.		

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